Transmission gain of multicarrier modulation

Anton Umek, Member, IEEE

Abstract— **The paper presents the results of research in the field of digital signal transmission on a metal twisted pair. The principal goal is to determine the information rate gain of multicarrier transmission system. In order to compare information rates we selected several transmission system models: coded and uncoded multicarrier and baseband transmission. A multicarrier transmission gain is explained also with a simple two-band channel example.**

Index Terms— **twisted pair, channel capacity , multicarrier modulation,**

I. Introduction

Unshielded twisted pairs are still used on a massive scale for short-distance transmission. The lower price of this transmission system, which is at present the main advantage of metal lines over optical lines, is essentially a result of the utilization of the existing cable infrastructure.

The paper deals with the theoretical transmission capacity and the technically available transmission capacity of a metal two-wire line in the vicinity of near-end crosstalk - NEXT. The model of a transmission channel is the same in all cases and corresponds to circumstances, which appear when the complete cancellation of intersymbol interference and echoes is given. To achieve this condition an ideal equalizer is needed as well as an ideal echo canceller. In practical conditions, however, it is sufficient when the residual noise of both adaptive filters is insignificant in comparison to NEXT.

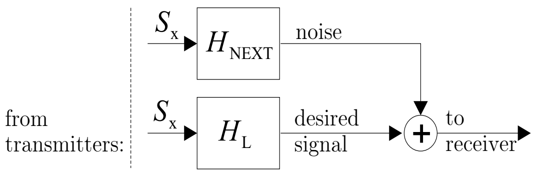

Fig. 1. SNR model.

II. Channel model and capacity

The NEXT channel model is given on figure 1. As many times before we will use simplified models for channel transfer function and near-end crosstalk.

The author is with the Faculty of Electrical Engineering at the University of Ljubljana, Slovenia, email:anton.umek@fe.uni-lj.si

Simplified so-called $\sqrt{f}$ attenuation model has a single parameter K_l, which is linearly proportional to line diameter. Propagation loss model is defined by equation:

$$|H_L(f)| = e^{-K_l l \sqrt{f}} \qquad (1)$$

A typical value $K_l = 2.3\,10^{-6}$ corresponds to 100 meters long UTP with attenuation 20dB at 100MHz.

Pair to pair crosstalk attenuation generally depends on quality of cable construction. NEXT signal is a sum of several crosstalk signals from closest adjacent pairs in the cable, which originates from independent sources positioned at the receiver side of the cable. Most often used NEXT power sum attenuation model defines a 1% worst-case NEXT power sum, which is assumed to fall 15 dB per decade with frequency [1]:

$$|H_{NEXT}(f)|^2 = K_{next} f^{\frac{3}{2}} \qquad (2)$$

Constant K_{next} also depends on the number of interfering adjacent pairs in the cable. Different values $K_{next} = 8.8\,10^{-14}$, $K_{next} = 1.7\,10^{-14}$ belongs to different subscriber loop NEXT models [2], [3]. Typical value of multipair crosstalk attenuation for modern standard UTP cables is much higher.

The signal to noise ratio at the receiver input is given from (1) and (2) by equation (3):

$$SNR(f) \approx 8.7 K_l\, l\, \sqrt{f} - 15 \log f - 10 \log K_{next} \qquad (3)$$

Signal to noise ration is rapidly decreasing with frequency and the whole transmission band over the equalization point $SNR = 0$dB is practical not useful.

Capacity of twisted pair transmission line in a crosstalk-limited environment has been already investigated by many authors [4], [5],[6]. In case that we do not have only a small number of interferers, we are allowed to assume that the probability distribution of NEXT signal is Gaussian. NEXT dominated channel capacity can be calculated as a sum of infinite number of infinitesimal narowband AWGN channels [7]:

$$R = \int_0^\infty \log_2\left(1 + \frac{|H_L(f)|^2}{|H_{NEXT}(f)|^2}\right) df. \qquad (4)$$

Figure 2 shows the results of numeric calculations by equation (4). More than 90 percent of the theoretical capacity R is reached up to frequency B_{0dB}, at which point the power spectrum density of noise and desired signal are equal.

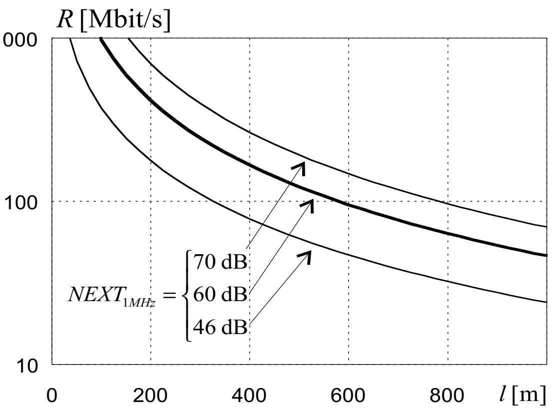

Fig. 2. NEXT channel capacity as a function of cable length.

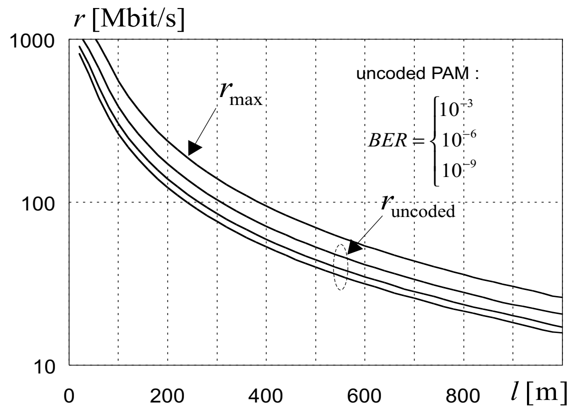

Fig. 5. Maximal information rate for a single-carrier modulation as a function of line length.

III. Single carrier and multicarrier transmission system

Beside the echo canceller and equalizer the level of technological sophistication of the equipment is also determined by selection of the modulation and coding methods. In order to compare available capacity we selected several hypothetical models of a transmission system: coded and uncoded baseband and multicarrier transmission.

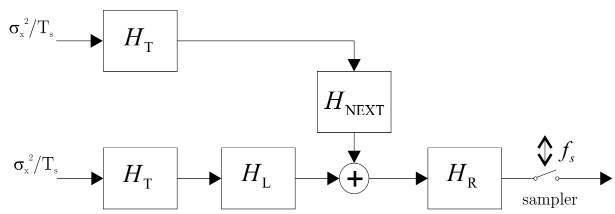

Fig. 3. Baseband transmission system model - PAM.

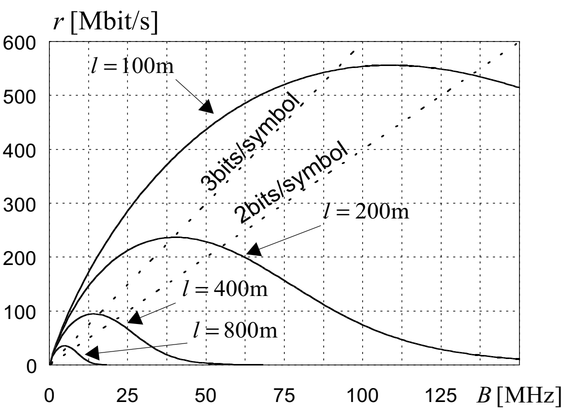

Fig. 4. Information rate for a single-carrier modulation as a function of bandwidth.

A. Ideal single carrier transmission

Figure 3 illustrates the model of single carrier transmission system. An impulse stream is feed through the transmitting filter to the channel input modeled with a linear transfer function $H_L(f)$. Received signal is a sum of distorted transmitted signal and noise (near-end crosstalk). The receiving filter is adjusted to minimize the effects of channel distortion. In order to minimize the effect of dispersion, the sampled overall transfer function $H_{TLR} = H_T H_L H_R$ should satisfy the first Nyquist criterion:

$$\sum_{k=-\infty}^{\infty} H_{TLR}(f + kf_s) = T_s \qquad (5)$$

Among infinite number of transfer functions that satisfies condition (5), we choose the ideal Nyquist filter with bandwidth equal to half symbol rate:

$$H_{TLR}(f) = \begin{cases} T_s\, e^{-\jmath 2\pi f t_0} & |f| \leq B = \frac{f_s}{2} \\ 0 & \text{elsewhere} \end{cases} \qquad (6)$$

Desired signal and NEXT signal are both passing the transmitting and the receiving filter. Therefore, it is not important how the joint function $H_{TR} = H_T H_R$ is divided between both filters. Desired signal and NEXT also have the same sources and therefore SNR remains unchanged by increasing the signal power. NEXT signal in the sampler input is assumed to have Gaussian distribution with zero mean and variance σ_n^2:

$$\sigma_n^2 = \sigma_x^2 T_s \int_{-\frac{f_s}{2}}^{\frac{f_s}{2}} \frac{|H_{NEXT}(f)|^2}{|H_L(f)|^2} df. \qquad (7)$$

Sampled received signal $y[i]$ is ISI free and distorted only by additive Gaussian noise:

$$y[i] = x[i-k] + n[i] \qquad (8)$$

The equivalent discrete channel model is therefore a Gaussian memoryless channel with capacity:

$$C = \frac{1}{2}\log_2(1 + \frac{\sigma_x^2}{\sigma_n^2}) \tag{9}$$

Information rate under assumption of ideal coding and decoding can be calculated as a product of discrete channel capacity and symbol rate:

$$r = Cf_s = B\log_2(1 + \frac{B}{\int_0^B \frac{|H_{NEXT}(f)|^2}{|H_L(f)|^2}df}) \tag{10}$$

Figure 4 shows the information rate curve (10) as a function of Nyquist bandwidth. The maximal rate (10) is achieved at optimal Nyquist channel bandwidth:

$$r(B_{opt}) = r_{max} \tag{11}$$

The maximal information rate for a single carrier modulation is shown as the top curve in Fig. 5.

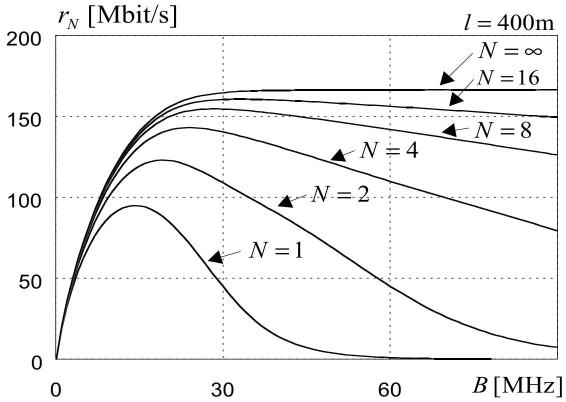

Fig. 6. MCM information rate as a function of bandwidth.

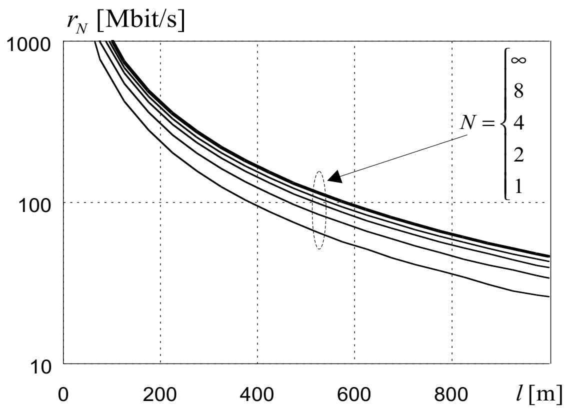

Fig. 7. MCM information rate as a function of cable length.

B. *Uncoded m-PAM*

The penalty for uncoded system is expressed as degradation of signal to noise ratio, known as β:

$$r_{uncoded} = B\log_2(1 + \frac{B}{\int_0^B \beta^2\frac{|H_{NEXT}(f)|^2}{|H_L(f)|^2}df}) \tag{12}$$

Figure 5 shows the maximal information rate for optimal level PAM as a function of BER and line length.

C. *Ideal multicarrier transmission*

Let as suppose, that we can perfectly divide a limited frequency band $(0, B)$ into a finite number N of non-overlapping channels with equal bandwidth:

$$\Delta f = \frac{B}{N} \tag{13}$$

A multicarrier information rate can be evaluated by adding together information rates of N perfectly equalized passband channels:

$$r_N(B) = \Delta f \sum_{n=1}^{N} \log_2(1 + \frac{\Delta f}{\int_{(n-1)\Delta f}^{n\Delta f} \frac{|H_{NEXT}(f)|^2}{|H_L(f)|^2}df}) \tag{14}$$

Figure 6 shows the multicarrier information rate distribution as a function of total bandwidth.

Results of numeric calculations of equation (14) for a different number of subchannels are shown in figure 7. Ideal single carrier modulation gives less than 60% of channel capacity. The greater part of information rate enhancement is done by using a small number of channels: four carriers provide approximately 86% and sixteen carriers over 96% of the theoretical capacity.

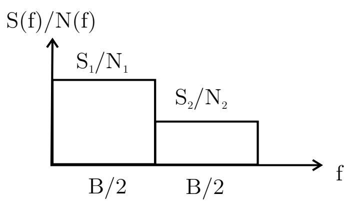

Fig. 8. Signal to noise profile.

IV. Transmission gain of multicarrier modulation

Transmisson gain of multicarrier modulation results from two effects:

- Capacity of a multicarrier system inside a limited frequency band is always greater than the capacity of a single carrier system, or equal if channel SNR is constant [6].

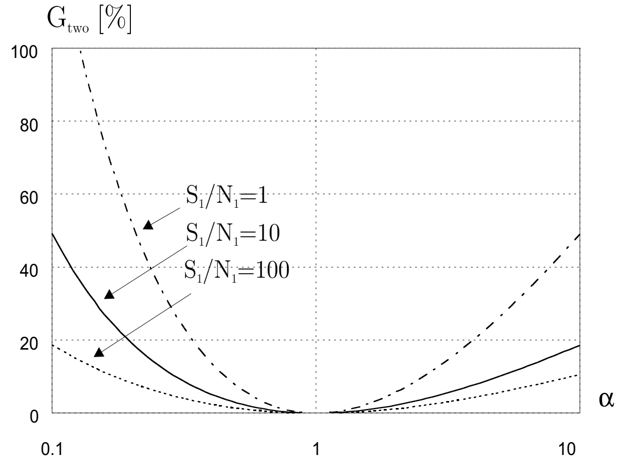

Fig. 9. Two-carrier vs single carrier capacity gain.

• The frequency band above optimal single-carrier bandwidth can be extra used by adding more frequency-separated channels.

A multicarrier system transmission gain can be easy explained, if we first study the system with only two carriers. Performance study is done for a hypotetical channel which gives a step signal to noise profile at the receiver input, as shown on figure 8. Signal to noise power density ratio in the left and right subband is generally not equal:

$$\frac{S_2}{N_2} = \alpha \frac{S_1}{N_1} \tag{15}$$

Transmission capacity of a two-carrier system r_{two} is the sum of information rates in two independent systems:

$$r_{two} = r_1 + r_2 \tag{16}$$

Under assumption of maximal symbol rate signalling and ideal coding the following equation is obtained:

$$r_{two} = \frac{B}{2}\log_2(1+\frac{S_1}{N_1}) + \frac{B}{2}\log_2(1+\frac{S_2}{N_2}) \tag{17}$$

Transmission rate of a single carrier system is calculated under assumption that the signal spectrum after equalization at the receiving filter output is white. The noise power at the receiving filter output is the sum of filtered noise in both subbands:

$$P_N = \frac{P_s}{2}(\frac{N_1}{S_1} + \frac{N_2}{S_2}) \tag{18}$$

Information bit rate of ideal single carrier system is thus:

$$r_{single} = B\log_2(1+\frac{2}{\frac{N_1}{S_1}+\frac{N_2}{S_2}}) \tag{19}$$

Expressions 17 and 19 are identical if signal to noise profile is flat ($\alpha = 1$). In other case $\alpha \neq 1$, the information rate of a two carrier system is greater than information rate of a single carrier system. In order to compare the information rates of both systems we can use a relative measure of a transmission gain:

$$G_{two} = 100(\frac{r_{two}}{r_{single}} - 1) \tag{20}$$

Figure 9 shows a two-carrier transmission gain for a group of channels with different signal to noise profiles.

V. Conclusions

The theoretical capacity of a two-wire line can be approached with the use of sophisticated coding methods and with transmission on frequency separated channels:

• Uncoded transmission means in the worst case, the loss of approximately half of the theoretical transmission capacity.

• The greater part of the theoretical transmission capacity is already achieved when a low number of carriers is used: four carriers provide approximately 86% of the theoretical capacity, sixteen carriers over 96% of the theoretical capacity.

A two-carrier transmission gain on hypothetic channel with a two-step SNR profile depends on absolute and relative SNR in both subbands. Illustrative results of numeric calculations for this case are collected on figure 9.

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