Gaussian pulse in Ultra-Wide Band communications

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Abstract—Ultra-Wideband (UWB) technology has been gaining increasing interest from the industry due to its recent application in communications. The technology itself has been well known for decades and its use was mostly reduced to radars in industrial applications. During the standardization process which is not over yet, two realizations were proposed – Impulse Radio (IR), which has been known for years and is synonymus with the UWB technology, and the recently unveiled UWB technology with OFDM (Orthogonal Frequency Division Multiplexing) modulation. We will focus on IR-UWB, which utilizes the pulses of duration in the order of nanoseconds yielding enormous bandwidth. UWB devices operate at limited power in an unlicensed spectrum. The efficiency of the system depends on the pulse shape. Since the optimal shape is not known, many pulse optimization methods have already been proposed. Gaussian pulse is one of the fundamental pulse shapes, therefore we made an analysis of the UWB system performance when using derivatives of Gaussian pulse. The capacity of the system depends on the order of the derivative, so the latter was percieved as the rule guideline evaluating the system performance.

Index Terms—Gaussian pulse, IEEE 802.15, Impulse Radio, Ultra-Wide Band.

1 Introduction

Ultra-Wideband (UWB) technology is gaining increasing interest from academia and industry for its potentials especially in the area of short-range indoor wireless communications [10]. UWB technology is also found in industrial applications such as imaging radars in medicine, radars used as navigation devices in cars, radars for searching people lost in natural and other disasters, etc.

The basic UWB operation is emitting pulses of a certain duration in the order of a nanoseconds directly to the antenna. It is a baseband operation, transceivers are thereby carrier-free and thus can be implemented with low complexity. Short pulse yields broad bandwidth, while the frequency spectrum shape depends on the shape of the pulse. UWB signal is defined as a signal having a fractional bandwidth (the ratio of baseband bandwidth to carrier frequency) greater than 0.2, or bandwidth greater than 500 MHz. UWB bandwidth is defined as a frequency band bounded by the points that are 10 dB below the highest radiated emission.

The typical spectrum of the UWB signal has a low amplitude and broad bandwidth. According to the amplitude, it would be theoretically possible to use UWB in a spectrum where some other devices are already in operation. In USA, UWB devices are allowed to operate at low power (an EIRP, Effective Isotropic Radiated Power, of –41.3 dBm/MHz) in an unlicensed spectrum from 3.1 GHz to 10.6 GHz with eventual out-of-band emissions that are at substaintally lower power levels. The spectrum band for UWB devices in Europe and Japan is not yet defined, the activities in this field, however, indicate that it will be the same as in USA.

The main standards activity of UWB technology takes place in IEEE 802.15.3a, an international standards work group which involves many of major companies (Time Domain, Xtreme Spectrum, Texas Instruments, Motorola etc.) [1]. The work group's main goal is to define the technical requirements and criteria for a physical layer of UWB technology with a low power consumption and low cost implementation. Two solutions were proposed during the standardization process: Impulse Radio (IR) and OFDM (Orthogonal Frequency Division Multiplexing) modulation.

Impulse Radio communicates with pulses of very short duration thereby occupying the spectrum of the radio signal from nearly 0 Hz to a few GHz. IR-UWB uses two separate channels – 3.1 GHz to 5.2 GHz and 5.8 GHz to 10.6 GHz. The band between 5.2 GHz and 5.8 GHz is not used by UWB in order to avoid the interefence with Wireless Local Area Networks IEEE 802.11a.

In the OFDM solution, the spectrum between 3.1 GHz and 10.6 GHz is divided into 528 MHz channels. Communication is thus less vulnerable to interference. The OFDM is a recent solution, while Impulse Radio is still synonymous with UWB.

In the remainder of the article, we focus on the properties of pulses in IR and their impact on IR-UWB performances. One of the most common pulse shape is Gaussian pulse, which is transformed by antenna into its first derivative called Gaussian monocycle. The inefficiency of such pulse is its broad and slowly decreasing spectral amplitude level. Such spectrum does not comply with the spectrum mask defined by regulation bodies. Many pulse optimisation methods can be found in literature ([10], [11]), but here we focus on Gaussian pulse and its derivatives. The order of the derivative impacts the shape, compliance with spectrum mask and capacity of the UWB communication system. The latter was the guideline rule in evaluating the properties of the pulses.

2 FREQUENCY AND POWER LIMITS

European standards in the field of UWB are in the domain of ETSI (European Telecommunications Standards Institute). The standards regulate communications, radiolocation and radionavigation, in case of the last two especially for the purposes of automotive industry. Standardization process named 802.15.3a lead by IEEE (Institute of Electrical and Electronics Engineers) is the most developed standard in the field of UWB.

According to the Shannon theorem, capacity of the communication channel grows with its bandwidth rather than Signal to Noise Ratio (SNR). Many companies wished to transmit UWB signals with the broadest possible bandwidth. U.S. Federal Communications Commission (FCC) allows the usage of the complete bandwidth (0 – 231 GHz) as long as EIRP does not exceed the levels defined by Part 15 [4]. ETSI shares the same opinion [2], while the problem arises in bands where some other low-power systems such as GPS (Global Positioning System) operate.

FCC Part 15 defines the maximum radiated power levels which electronic devices may radiate not to interfere with other devices and to be proof against the other devices' interference. Wideband communications are specific, because the maximum power levels may not be the same in the whole bandwidth. This is crucial near the bands used by some special services (such as radionavigation, etc.).

Maximum EIRP in the band between 3.1 GHz and 10.6 GHz is limited to –41.3 dBm/MHz. This rule was set by FCC while ETSI proposed very similar limits, as depicted in Figure 1 [2].

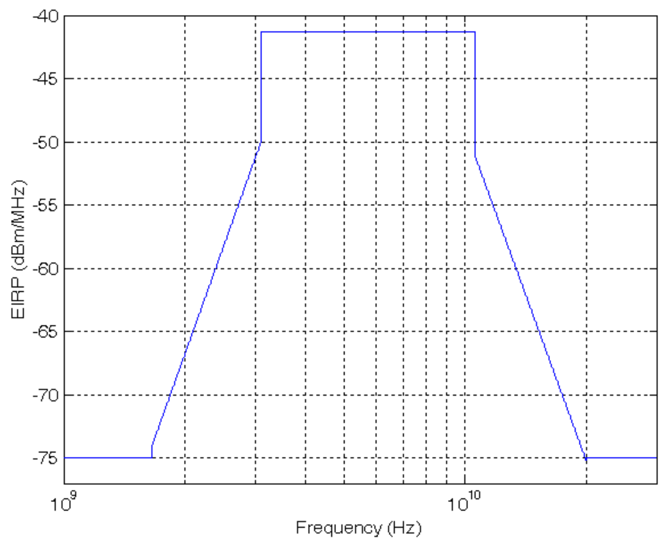

Fig. 1. spectrum mask for UWB communications as proposed by ETSI.

3 GAUSSIAN IMPULSE ANALYSIS

For decades, Impulse Radio (IR) has been synonymus with forming all kinds of UWB signals which also holds true for UWB communications [9]. IR is based on sequential transmission of pulses of very short duration. In turn, short pulses are viewed as spanning frequency band with enormous bandwidths. A single user transmits one pulse during the time of one time-frame (T_f – frame time). The position of

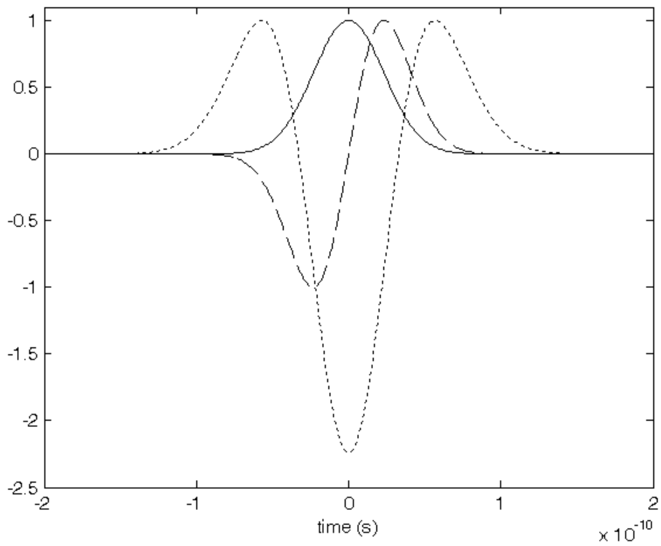

Fig. 2. Gaussian pulse (solid line), 1st derviative of Gaussian pulse (dashed line) and 2nd derivative of Gaussian pulse (dotted line).

single user's pulse inside the time frame is variable (position modulation). The spectral properties of the UWB signal in the radio channel depend on the shape, time duration of the pulse and properties of the antenna. The frequency response of such a wideband antenna might vary, which can be a serious problem.

Gaussian, Rayleigh and Laplacian pulses are among the most popular pulses in UWB. Gaussian pulse in time domain is modeled by equation 1.

$$w(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} \quad (1)$$

σ represents the standard deviation and μ the mean value. Stating μ=0, the spectrum of Gaussian pulse is:

$$W(f) = \frac{1}{2\sigma^2} e^{-\frac{1}{2}(2\pi\sigma f)^2} \quad (2)$$

Since the antenna acts as a differentiator [6], the shape of the pulse in the radio channel is modeled as first order derivative of (1). Its frequency spectrum can be estimated using the properties of the Forurier transform which states that the differentiation in time domain is equivalent to multiplying with frequency (f) in frequency domain. Central frequency of n-th order derivative of Gaussian pulse is:

$$f_{(n)} = \frac{\sqrt{n}}{2\pi\sigma} \quad (3)$$

As Figure 2 depicts, first order of Gaussian pulse derivative does not comply with the spectrum mask defined by

ETSI or FCC. Because of the spectrum being too wide, the power level should be reduced to satisfy regulations. One can also notice that the higher order derivatives require less power reduction, while the 4th and higher dervatives comply with the spectrum mask.

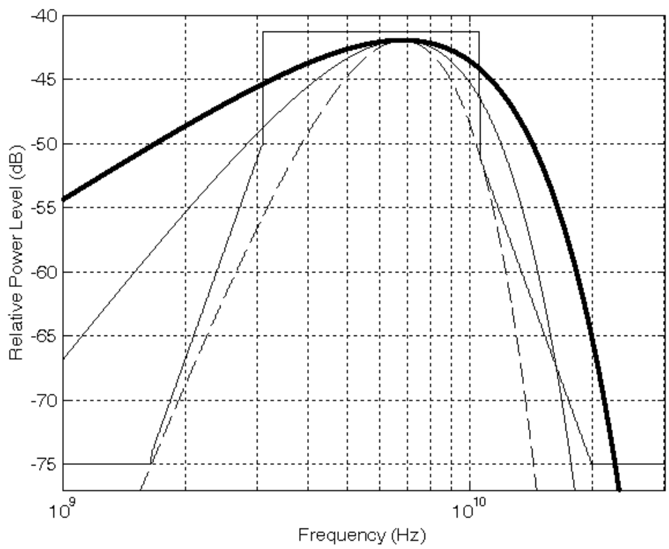

Fig. 3. Spectrum mask as propossed by ETSI (angular solid line), 1st derivative of Gaussian pulse (bold line), 2nd derivative (solid line) and 4th derivative (dashed line).

4 CAPACITY OF UWB SYSTEM IN DEPENDENCE OF PULSE SHAPE

4.1 UWB properties

The maximum number of active users in IR-UWB system roughly depends on the shape of pulse ($w(t)$), value of position modulation parameter (δ), level of interference and Signal to Noise Ratio (SNR).

The postion modulation receiver correlates the received signal with the sample $v(t)$, which is a combination of two basic waveforms $w(t)$:

$$v(t) = w(t) - w(t-\delta) \tag{4}$$

Figure 4 depicts pulse $w(t)$ and sample $v(t)$. One can see how $v(t)$ depends on the parameter δ which should therefore be appropriately choosen to give the best results. Since correlation represents the multiplication of two signals, one can also see the result no matter whether bit 0 or 1 is transmitted. It is also possible to see the impact of the parameter δ on the result.

Signal to noise ratio for a single user in environment with N_u active users is defined as [9]:

$$SNR(N_u) = \frac{(N\ A_1 m_p)^2}{\sigma^2 + N\ \sigma_a^2 \sum_{k=2}^{N_u} A_k^2} \tag{5}$$

Noise is a sum of thermical noise and other user's interference which is modeled by Gaussian noise. σ^2 is the variance of the receiver noise, N is the number of pulses needed to transmit one symbol of information, σ_a^2 is interference caused by other active users and m_p represents the correlation between the sample waveform and single user's pulse. Correlation corresponds to the useful energy (m_p^2). Parameters σ_a and m_p depends on the pulse shape and parameter of position modulation δ.

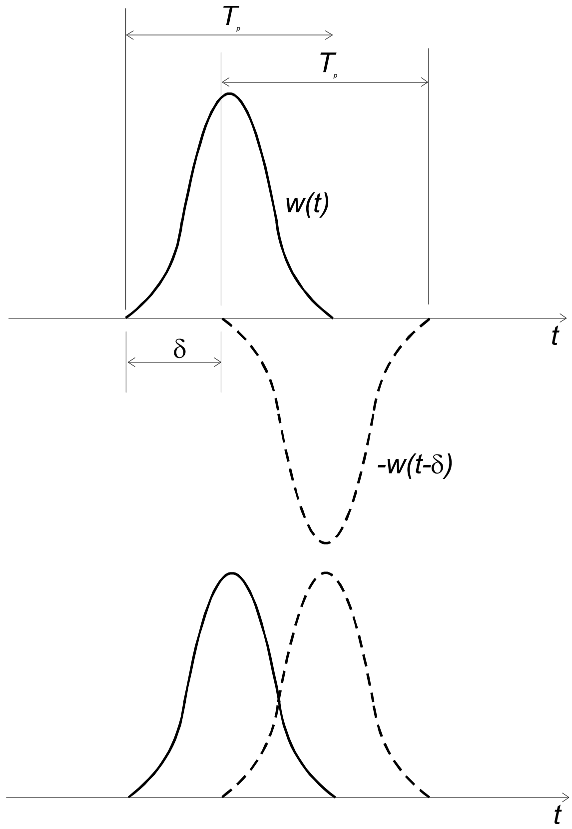

Fig. 4. position modulation and its parameter.

$$m_p = \int_{-\infty}^{\infty} w(x-\delta)v(x)dx \tag{6}$$

Interference energy is the product of the number of receiver inputs, amplitude characteristics of transmission path and average power caused by non-synchronized pulse of some other user :

$$\sigma_a^2 = \frac{1}{T_f} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} w(x-s)v(x)dx \right]^2 ds \tag{7}$$

Signal to noise ratio of the impulse radio can be rewritten as:

$$SNR(N_u) = \left[\frac{1}{SNR(1)} + M \sum_{k=2}^{N_u} \left(\frac{A_k}{A_1} \right)^2 \right]^{-1} \tag{8}$$

where M is:

$$M^{-1} = \frac{N\ m_p^2}{\sigma_a^2} \tag{9}$$

The equations above hold for any number of users.

Suppose that a minimum signal to noise ratio SNR_{min} must be maintained for each user to satisfy a performance specification when N_u-1 other users are active. Ratio of SNR_{min} to *SNR*, *SNR(1)*, represents the increase in every transmitter's power that is required to maintain the specified error probability at the output of the receiver. The increase in the required power (in units of dB) is defined as: $\Delta P = 10\log[SNR(1)/SNR_{min}]$.

Under the assumption of perfect power control, which is necessary for the reduction of the interference, the theoretical maximum number of active users of the IR-UWB system is:

$$N_{max} = \lfloor M^{-1} SNR_{min}^{-1} \rfloor + 1 \qquad (10)$$

In (10), *SNR* is in linear units. One can see that the number of users at the specified bit error rate (BER) and specified SNR_{min} cannot be greater than N_{max}, regardless of the additional power.

In the case of single user in the system, the receiver's noise dominates the multiple-access noise and is therefore considered as the only interferer. The optimum choice of the position modulation parameter δ in that case is the one that maximizes $|m_p|$. On the other hand, when there is a lot of active users in the system, the receiver's noise is negligible, therefore the optimum choice of δ is the one that maximizes $|m_p|/\sigma$.

4.2 UWB system capacity

According to (9), the maximum number of active users in the IR-UWB system is defined with *SNR* and *M*. To maintain bit error rate BER=10^{-5} in a communications system with no error control coding, $SNR_{min} = 13dB$ must be obtained. This is true in the case of relatively good demodulator in the receiver. For BER=10^{-6} $SNR_{min} = 14dB$ is required and $SNR_{min} = 10dB$ for $BER=10^{-3}$. Parameter *M* (in eq. 10) depends on the pulse shape and position modulation parameter δ.

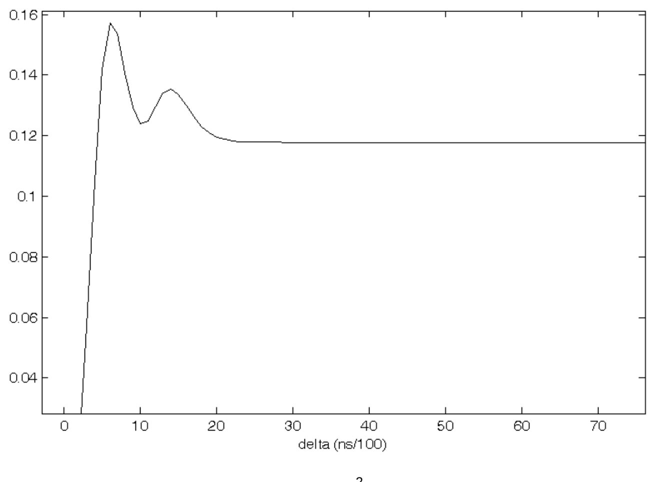

Fig. 5. Maximizing parameter $(|m_p|/\sigma_a)^2$.

We searched for the optimum of the position modulation parameter δ numerically. In case of the first derivative of Gaussian pulse with the central frequency f_C = 6.85GHz ($\sigma = 2.32 \cdot 10^{-11}, T_p = 0.15ns, \mu = 0$), the result of optimum searching is depicted in figure 5 (constant factors were omitted during the analysis).

The optimal value of the parameter δ in the above example is 0.06 ns. Considerring δ, we can estimate $(|m_p|/\sigma_a)^2$ = 1573. According to the specified SNR_{min}, we can find out the number of the active users in the system. First, we need *N* – number of pulses needed to transmit one symbol of information. *N* is related to data rate $R_d = 1/NT_f$ (T_f – time frame duration). Let's suppose $R_d = 1Mbit/s$, then $T_f = 100ns \Rightarrow N = 10$. If bit error rate is 10^{-6}, then $SNR_{min} = 14dB$ and N_{max} = 630.

As it was already mentioned above, the main problem with low order derivatives of the Gaussian pulse is relatively broad spectrum which does not comply with spectrum masks defined regulatory bodies and therefore needs to be attenuated before being emitted into the radio channel. The higher order derivatives of the Gaussian pulse have narrower spectra in comparison to the low order derivatives and therefore need less attenuation. On the other hand, the number of maximum active users in the system falls when the order of the derivative rises.

The signals used in our experiments should work in real applications and therefore should comply with the spectrum masks defined by regulatory bodies. The central frequency of each test signal was set to 6.85 GHz which is the aritmetic mean of UWB spectrum mask's bandwidth. Table 1 shows the results of the typical IR-UWB system properties in dependence to different order derivatives of Gaussian pulse. In all cases we stated $SNR_{min} = 14dB$.

derivative	δ (ns)	N_{max}	*A* (dB)
x'(t)	0.06	630	25
$x^{(2)}$ (t)	0.07	514	17
$x^{(3)}$(t)	0.07	448	8
$x^{(4)}$(t)	0.08	406	0
$x^{(5)}$(t)	0.09	317	0
$x^{(6)}$(t)	0.08	351	0
$x^{(7)}$(t)	0.08	331	0

Table 1: IR-UWB propertis if using Gaussian pulse derivatives.

When higher order derivatives of Gaussian pulse are used, transceivers can emit more power into the radio channel as was described above. Commom attenuation of 41.3 *dBm* is required for each UWB signal to satisfy regulations. Additional attenuations for the low order derivaties of Gaussian pulse are showed in table 1 where it is also showed that number of active users decreases.

In the above examples central frequency of each test signal was set to 6.85 GHz. In the cases where large attenuation is required, pulse spectra would fit the spectrum mask with less attenuation if the central frequency is modified. The result of such modification for the first order of Gaussian pulse is depicted in figure 6.

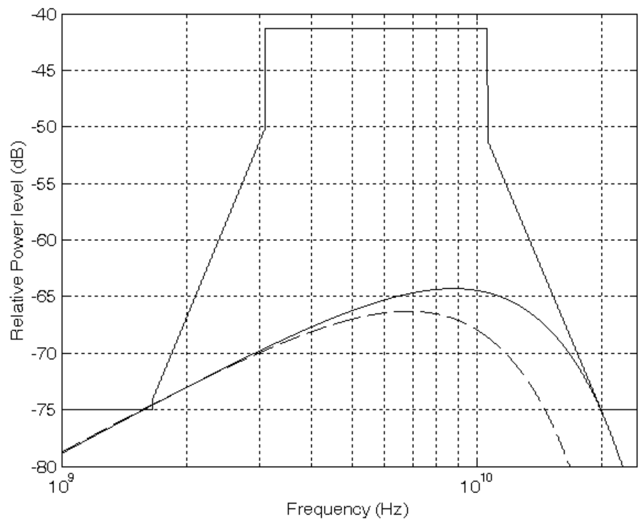

Fig. 6. The first order derivative of Gaussian pulse with f_c=6.85 GHz (dashed line), and with f_c=8.8 GHz (solid line).

Modification of the central frequency impacts other parameters of IR-UWB communication system as well. As we can conclude from the results in table 1, modification of central frequency and attenuation is necessary for the first three orders of the derivatives. How the communication system performances change is shown in table 2.

derivative	f_c (GHz)	δ (ns)	N_{max}	A (dB)
x'(t)	8.8	0.05	838	23
$x^{(2)}$ (t)	8.2	0.06	624	13.5
$x^{(3)}$(t)	7.4	0.07	490	6

Table 2: IR-UWB propertis if using the low order Gaussian pulse derivatives with optimized central frequency.

The choice of the optimal waveform is therefore always a trade-off between the emitted power and the possible number of users and the specific characteristics of a single application prove which feature would predominate.

5 Conclusion

The impact of pulse shape on the performance of IR-UWB communication system was investigated in this article. The discussion concentrates on the pulse shapes gained by differentiating the Gaussian pulse to find out how the maximum number of active users depends on the order of the derivative. It was established that low order derivatives are to broadband to meet the spectrum masks defined by the regulatory bodies and can only be used in practical applications if attenuated. In order to preserve the maximum power, many different optimization methods were already suggested. Since the maximum EIRP defined by regulatory bodies (FCC, ETSI etc.) is –41.3 dBm/MHz, one can see that additional attenuation would not be desirable. Taking into consideration only the spectrum, higher order derivatives comply with the spectrum masks much better.

The main purpose of this article was to check the possibilities of using Gaussian pulse derivatives in IR-UWB communication systems. With regard to spectral properties, only higher order derivatives are appropriate, while on the other hand, the maximum number of active users decreases with the increasing order of derivative. The optimal pulse shape choice is therefore a compromise based on the needs for every individual case.

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